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COMMENT

Interdimensional scaling laws for the critical exponents of the percolation threshold

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Abstract. The interdimensional scaling laws of second-order phase transitions are modified so as to apply to the percolation threshold. Interesting results are obtained for some of the geometric exponents as well as for some conductivity exponents in the case of a metal–insulator or superconductor–normal conductor composite or a random resistor network.

Interdimensional scaling laws (ISL) that connect critical exponents in similar systems but with different dimensionalities were first derived some years ago in the context of a second-order phase transition (Imry *et al* 1973, Bergman *et al* 1973). Starting from the exactly known exponent of the 2D (two-dimensional) Ising model, the ISL gave values for the exponents of the 3D Ising model which, though not nearly as good as the more systematic modern approximation methods, were nevertheless quite respectable, i.e. $\nu_3=\frac{2}{3}$, $\gamma_3=\frac{5}{4}$, $\beta_3=\frac{3}{8}$ as compared with the best numerical values $\nu_3=0.63$, $\gamma_3=1.24$, $\beta_3=0.32$.

We recall that the ISL were obtained by considering a system which is finite in δ of its total number of dimensions d, and by comparing the transition from d- to $(d-\delta)$-dimensional behaviour with the transition from mean-field to critical behaviour. Because the latter transition, which defines the Ginzburg critical region, depends on the critical dimensionality d_c, we have to modify appropriately the ISL of Bergman *et al* (1973) in order to apply them generally, and to percolation in the present case. For the critical behaviour of a general quantity

$$X(\varepsilon)\propto|\varepsilon|^{-x_d} \qquad \text{in } d \text{ dimensions}, \qquad \varepsilon\equiv p-p_c, \tag{1}$$

where p is the volume fraction of the percolating component and p_c is the percolation threshold, we thus obtain

$$\frac{\chi_{d-\delta}-\chi_d}{\nu_d}=\frac{\chi_{d-\delta}-\chi_m}{\nu_m}\,\frac{\delta}{d_c-d+\delta}, \tag{2}$$

where ν is the correlation length exponent and the subscript m denotes the mean-field values (i.e. the values at $d=d_c$). While this is, of course, an approximate relationship, the reader can easily convince himself that it is exact to order d_c-d and δ when both these quantities are small. It should also be noted that (2) preserves all the linear scaling relationships (see Imry *et al* 1973), e.g.

$$2\beta=d\nu-\gamma_p, \tag{3}$$

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so that results obtained for β are not independent of the results for ν and γ_p. We should also point out that the derivation of (2) is valid only if the following inequality holds:

$$\delta\nu_d < (d_c - d + \delta)/2, \qquad (4)$$

which is indeed true for the case considered here where $d_c = 6$, $d = 3, 4, 5$, and $\delta = d - 2$.

In order to deduce some numerical consequences from (2) for the percolation and conductivity exponents, we first use this equation for the case $\chi \equiv \nu$ to obtain ν_d from the supposedly known value of $\nu_2 = \frac{4}{3}$. We then use this value of ν_d to obtain other critical exponents χ_d from the supposedly known value of χ_2.

The 2D exponents we used include the apparently exact values for ν_2, γ_{p2}, β_2 due to den Nijs (1979), Black and Emery (1981) and Nienhuis (1982), the exact value for g_2 (the divergence exponent of the Hall coefficient) due to Shklovskii (1977), the average numerical value for $t_2 = s_2$ (t is the exponent that characterises the vanishing of the conductivity in a metal–insulator composite as $p \to p_c^+$ while s is the exponent that characterises the divergence of the conductivity in a superconductor–normal conductor composite as $p \to p_c^-$) due to Binder and Stauffer (1983), the numerical value for ζ_2 (the divergence exponent of the resistance between two points on the percolating cluster separated by an aerial distance equal to the correlation length ξ) due to Fisch and Harris (1978), and the numerical value for β_{B2} (the divergence exponent for the backbone volume) due to Kirkpatrick (1978). The results for all these exponents when $d = 3$, 4, 5 are shown in table 1 together with the input values at 2D and at 6D (the mean field values). For comparison, we show in table 2 a sample collection of numerical values for the same exponents in $d = 3$, 4, 5.

When the ISL results for ν, γ_p, t are compared with the numerical results, it is clear that the values are all in good agreement. We also note that our result for t_3, while in reasonable agreement with the value 1.95 of Fisch and Harris (1978), is in strong disagreement with the widely accepted value 1.70 (Straley 1977). Interestingly, a new calculation of t_3 seems to obtain values that are even closer to our result, namely

Table 1. Calculated values of critical geometric and conductivity exponents for $d = 3$, 4, 5 from interdimensional scaling laws (ISL). Input data are the mean-field ($d = 6$) values, and the $d = 2$ values. Note that the values for β are not independent of the values for ν and γ_p as a consequence of equation (3). Although some of the results for $d = 3$, 4, 5 are expressible as exact rational fractions, we present them in decimal form in order to facilitate comparison with the numerical results in table 2.

Exponent	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$
β_B	0.55[a]	1.23	1.60	1.84	2[f]
β	5/36[b]	0.54	0.77	0.90	1
ν	4/3[b]	0.94	0.73	0.59	1/2
γ_p	43/18[b]	1.74	1.38	1.15	1
ζ	1.43[c]	1.23	1.12	1.05	1[c]
t	1.28[d]	2.09	2.53	2.81	3
s	1.28[d]	0.68	0.35	0.14	0
g	0[e]	0.47	0.73	0.89	1[g]

[a] Kirkpatrick (1978). [b] Nienhuis (1982). [c] Fisch and Harris (1978). [d] Binder and Stauffer (1983). They determine this value as an average of a number of independent calculations by different authors. [e] Shklovskii (1977). [f] Gefen *et al* (1981). [g] Straley (1980a).

Table 2. Numerical values for some of the exponents of table 1 at $d = 3, 4, 5$ from various sources which are cited below. The values for β were obtained from the values for ν and γ_p by using equation (3).

Exponent	$d = 3$	$d = 4$	$d = 5$
β_B	0.9[a]	1.1[a]	—
β	0.45[b]	0.62[b]	0.84[b]
ν	0.88[c]	0.66[d]	0.57[d]
γ_p	1.74[c]	1.40[d]	1.17[d]
ζ	1.12[d]	1.05[d]	1.02[d]
t	1.95[d]	2.37[d]	2.73[d]
s	0.70[e]	—	—
g	0.3[f]	—	—

[a] Kirkpatrick (1978). [b] Equation (3). [c] Heermann and Stauffer (1981). Slightly different values for γ_{p3} were recently obtained by Gaunt and Sykes (1982) and Margolina *et al* (1983). [d] Fisch and Harris (1978). [e] Straley (1977). [f] Bergman *et al* (1983).

2.04 for a simple cubic random-bond network and 2.15 for a simple cubic random-site network (Mitescu and Greene 1983). Our result for the Hall exponent g_3 is compared in table 1 with a value recently obtained by simulating the Hall effect on a special type of random resistor network (Bergman *et al* 1983). It may also be compared with the nodes–links picture estimate $g_3 = 0.5$–0.6 (Straley 1980b).

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